Transversal extensions of transversal matroids

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Joint work with:
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What is it about?

- Take a transversal matroid $M$ on the ground set $E$
- Let $N$ be a transversal matroid on $E \cup \{x\}$ such that $N \setminus \{x\} = M$
- We would like to know about $N$
What is it about?

- Take a transversal matroid $M$ on the ground set $E$.
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- We would like to know about $N$.

For instance:
- How do we obtain one/all such $N$?
- Are there many such $N$’s?
- How do different $N$’s relate to each other?
Warm up

Let $E$ be a finite set. Do the following:

- Take a collection $A_1, \ldots, A_r$ with $A_i \subseteq E$, $1 \leq i \leq r$

Claim: \{e_1, \ldots, e_k\} is affinely independent if there are $i_1, \ldots, i_k$ all different such that $e_j \in A_{i_j}$ for $1 \leq j \leq k$ (it follows from Hall's theorem)
Warm up

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- Take a collection $A_1, \ldots, A_r$ with $A_i \subseteq E$, $1 \leq i \leq r$
- Take an $r$-simplex $\Delta$ with vertices $v_1, \ldots, v_r$

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- Take an $r$-simplex $\Delta$ with vertices $v_1, \ldots, v_r$
- For each $e \in E$, place $e$ on the face spanned by $\{v_i : e \in A_i\}$ as freely as possible

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(it follows from Hall’s theorem)
Transversal matroids

Let $A_1, \ldots, A_r$ be subsets of a finite set $E$

A subset $\{e_1, \ldots, e_k\} \subseteq E$ is a partial transversal of $A_1, \ldots, A_r$ if there are $i_1, \ldots, i_k$ all different such that $e_j \in A_{i_j}$ for all $1 \leq j \leq k$

**Thm** (Edmonds and Fulkerson 1965)
The partial transversals of $A_1, \ldots, A_r$ are the independent sets of a matroid on $E$
Def  A matroid consists of
- a finite non-empty set $E$ (the ground set)
- a family $\mathcal{I}$ of subsets of $E$ (the independent sets)

such that

1.1 $\emptyset \in \mathcal{I}$

1.2 if $I' \subseteq I \in \mathcal{I}$ then $I' \in \mathcal{I}$

1.3 if $I_1, I_2 \in \mathcal{I}$ and $|I_1| < |I_2|$, then there exists $e \in I_2 \setminus I_1$ such that $I_1 \cup e \in \mathcal{I}$

Example: $E$ a finite set of points in affine space $I = \{ I \subseteq E : I$ is affinely independent $\}$
**Matroids**

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**Example:**
$E$ a finite set of points in affine space
$\mathcal{I} = \{ I \subseteq E : I \text{ is affinely independent} \}$
Some matroid facts

- Given $X \subseteq E$, all maximal independent sets contained in $X$ have the same size, the *rank* $r(X)$ of $X$
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- If $x \in X$ is such that $r(X - x) = r(X) - 1$, we say that $x$ is a *coloop* of $X$ (so $x$ is in all maximal independent sets of $X$)
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- Given $M$ and an element $e \in E$, the deletion $M \setminus e$ is the matroid on $E - e$ with independent sets $\{I \in \mathcal{I} : e \not\in I\}$
A transversal matroid

\[ \mathcal{A} = (\{1, 2, 5, 6, 7\}, \{3, 4, 5, 6, 8\}, \{7, 8\}) \] gives

\begin{center}
\begin{tikzpicture}
\fill (0,0) circle (4pt);
\fill (1,0) circle (4pt);
\fill (2,0) circle (4pt);
\fill (3,0) circle (4pt);
\fill (4,0) circle (4pt);
\fill (5,0) circle (4pt);
\fill (6,0) circle (4pt);
\fill (7,0) circle (4pt);
\fill (8,0) circle (4pt);
\draw (0,0) -- (1,0);
\draw (1,0) -- (2,0);
\draw (2,0) -- (3,0);
\draw (3,0) -- (4,0);
\draw (4,0) -- (5,0);
\draw (5,0) -- (6,0);
\draw (6,0) -- (7,0);
\draw (7,0) -- (8,0);
\end{tikzpicture}
\end{center}
A transversal matroid

\[ A = (\{1, 2, 5, 6, 7\}, \{3, 4, 5, 6, 8\}, \{7, 8\}) \] gives

Observe that \((\{1, 2, 5, 6, 7\}, \{3, 4, 5, 6, 7\}, \{7, 8\})\) also gives the same matroid
A transversal matroid

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Observe that \((\{1, 2, 5, 6, 7\}, \{3, 4, 5, 6, 7\}, \{7, 8\})\) also gives the same matroid

The collections \((\{1, 2, 5, 6, 7\}, \{3, 4, 5, 6, 8\}, \{7, 8\})\) and \((\{1, 2, 5, 6, 7\}, \{3, 4, 5, 6, 7\}, \{7, 8\})\) are presentations of the transversal matroid
**Thm** (Brylawski 1975) “Transversal matroids are those that admit a representation in a simplex $\Delta$ where elements lie on the faces of $\Delta$ is the most free possible way”

Given such a representation, we can recover the presentation as

$$A_i = \{ x : x \text{ is not on the face opposite to vertex } v_i \}$$
Example again

\[ A = (\{1, 2, 5, 6, 7\}, \{3, 4, 5, 6, 8\}, \{7, 8\}) \]
Example again

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The poset of all presentations
Some facts about presentations

The set of presentations of a transversal matroid is ordered by set inclusion

- A transversal matroid can typically have many minimal presentations

- The sets in a minimal presentation are cocircuits (Las Vergnas 70, Bondy and Welsh 71)
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- A transversal matroid can typically have many minimal presentations

- The sets in a minimal presentation are cocircuits (Las Vergnas 70, Bondy and Welsh 71)

- A transversal matroid has a unique maximal presentation (Mason 69, Bondy 72)

- If \((A_1, A_2, \ldots, A_r)\) and \((B_1, A_2, \ldots, A_r)\) are presentations with \(A_1 \subset B_1\), the elements of \(B_1 \setminus A_1\) are coloops of \(M \setminus A_1\) (Bondy and Welsh 71)
Transversal extensions

A (single-element) extension of a matroid $M$ on $E$ is a matroid $N$ on $E \cup \{x\}$ such that $M = N \setminus \{x\}$ (and, for us, $r(N) = r(M)$)

- The theory of extensions is well-understood: extensions of $M$ are in bijection with some families of subsets called “modular cuts of flats” (Crapo 65)
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- If $(B_1, \ldots, B_r)$ is a presentation of $N$, then $(B_1 \setminus x, \ldots, B_r \setminus x)$ is a presentation of $M = N \setminus x$
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- So all transversal extensions of $M$ can be obtained by adding $x$ to some sets in some presentation of $M$. 
Transversal extensions

A (single-element) extension of a matroid $M$ on $E$ is a matroid $N$ on $E \cup x$ such that $M = N \setminus x$ (and, for us, $r(N) = r(M)$)

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A transversal extension of a transversal matroid $M$ is an extension of $M$ that is also transversal

- If $(B_1, \ldots, B_r)$ is a presentation of $N$, then $(B_1 \setminus x, \ldots, B_r \setminus x)$ is a presentation of $M = N \setminus x$
- So all transversal extensions of $M$ can be obtained by adding $x$ to some sets in some presentation of $M$.
- But could it be that we get repetitions? How can we ensure we have all extensions?
Notation

Let $\mathcal{A} = (A_1, \ldots, A_r)$ be a presentation of $M$. For $I \subseteq [r]$, let

$$\mathcal{A}' = \begin{cases} 
A_i \cup x, & \text{if } i \in I, \\
A_i, & \text{otherwise}.
\end{cases}$$

The matroid $M[\mathcal{A}']$ is a transversal extension of $M$. 
Notation

Let $\mathcal{A} = (A_1, \ldots, A_r)$ be a presentation of $M$
For $I \subseteq [r]$, let

$$\mathcal{A}' = \left\{ \begin{array}{ll}
A_i \cup x, & \text{if } i \in I, \\
A_i, & \text{otherwise.}
\end{array} \right. $$

The matroid $M[\mathcal{A}']$ is a transversal extension of $M$

Example: $\mathcal{A} = (\{1, 2, 5, 6, 7, 8\}, \{3, 4, 5, 6, 7\}, \{7, 8\})$

$I = \{1, 2, 3\}$
Notation

Let $\mathcal{A} = (A_1, \ldots, A_r)$ be a presentation of $M$

For $I \subseteq [r]$, let

$$\mathcal{A}' = \begin{cases} 
A_i \cup \{v\}, & \text{if } i \in I, \\
A_i, & \text{otherwise.}
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A_i \cup x, & \text{if } i \in I, \\
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The matroid $M[\mathcal{A}']$ is a transversal extension of $M$.

Example: $\mathcal{A} = (\{1, 2, 5, 6, 7, 8\}, \{3, 4, 5, 6, 7\}, \{7, 8\})$

$I = \{2, 3\}$
Notation

Let $\mathcal{A} = (A_1, \ldots, A_r)$ be a presentation of $M$
For $I \subseteq [r]$, let

$$\mathcal{A}' = \begin{cases} 
    A_i \cup x, & \text{if } i \in I, \\
    A_i, & \text{otherwise}.
\end{cases}$$

The matroid $M[\mathcal{A}']$ is a transversal extension of $M$

Example: $\mathcal{A} = (\{1, 2, 5, 6, 7, 8\}, \{3, 4, 5, 6, 7\}, \{7, 8\})$

$I = \{3\}$
Notation

Let \( \mathcal{A} = (A_1, \ldots, A_r) \) be a presentation of \( M \)
For \( I \subseteq [r] \), let

\[
\mathcal{A}' = \left\{ \begin{array}{ll}
A_i \cup x, & \text{if } i \in I, \\
A_i, & \text{otherwise}.
\end{array} \right.
\]

The matroid \( M[\mathcal{A}'] \) is a transversal extension of \( M \)

Example: \( \mathcal{A} = (\{1, 2, 5, 6, 7, 8\}, \{3, 4, 5, 6, 7\}, \{7, 8\}) \)

\( I = \{1, 2\} \)
Only minimal presentations give different extensions

**Thm 1**
The following are equivalent:

(i) if $I \neq J$ then $M[A^I] \neq M[A^J]$

(ii) the presentation $A$ is minimal

**Cor** If $A$ is a minimal presentation of $M$, then $A^I$ is a minimal presentation of $M[A^I]$
Minimal presentations give all possible extensions

**Thm 2**
If $N$ is a transversal extension of $M$, there exist a minimal presentation $\mathcal{A}$ of $M$ and a set $I \subseteq [r]$ such that $N = M[\mathcal{A}^I]$
Minimal presentations give all possible extensions

**Thm 2**
If $N$ is a transversal extension of $M$, there exist a minimal presentation $\mathcal{A}$ of $M$ and a set $I \subseteq [r]$ such that $N = M[\mathcal{A}^I]$

Unfortunately we do not know how to get all minimal presentations of all extensions...
Proof of Thm 1 (I)

**Thm 1** The following are equivalent:

(i) if \( I \neq J \) then \( M[A^I] \neq M[A^J] \)

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Proof of (ii) \( \Rightarrow \) (i)
Proof of Thm 1 (I)

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Proof of (ii) $\Rightarrow$ (i)

As $A$ is minimal, the $A_i$ are cocircuits, so $H_i = E - A_i$ are hyperplanes of $M[A]$:
**Proof of Thm 1 (I)**

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As $\mathcal{A}$ is minimal, the $A_i$ are cocircuits, so $H_i = E - A_i$ are hyperplanes of $M[\mathcal{A}]$:

- If $i \in I$, then $H_i$ is a hyperplane of $M[A^I]$
Proof of Thm 1 (l)

**Thm 1** The following are equivalent:

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(ii) the presentation \( \mathcal{A} \) is minimal

Proof of (ii) \( \Rightarrow \) (i)

As \( \mathcal{A} \) is minimal, the \( A_i \) are cocircuits, so \( H_i = E - A_i \) are hyperplanes of \( M[\mathcal{A}] \):

- If \( i \in I \), then \( H_i \) is a hyperplane of \( M[A^I] \)
- If \( i \not\in I \), then \( H_i \cup \mathcal{X} \) is a hyperplane of \( M[A^I] \)
**Proof of Thm 1 (i)**

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Proof of (ii) ⇒ (i)

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- If \( i \in I \), then \( H_i \) is a hyperplane of \( M[\mathcal{A}^I] \)
- If \( i \not\in I \), then \( H_i \cup x \) is a hyperplane of \( M[\mathcal{A}^I] \)

So looking which \( H_i \) are hyperplanes in \( M[\mathcal{A}^I] \) we can recover \( I \)
Thm 1  The following are equivalent:

(i) if $I \neq J$ then $M[A^I] \neq M[A^J]$

(ii) the presentation $\mathcal{A}$ is minimal

Idea of proof of (i) $\Rightarrow$ (ii)
Proof of Thm 1 (II)

Thm 1 The following are equivalent:

(i) if $I \neq J$ then $M[A^I] \neq M[A^J]$

(ii) the presentation $A$ is minimal

Idea of proof of (i) $\Rightarrow$ (ii)

Assume $A$ is not minimal. Say $A_r$ is not a cocircuit
Proof of Thm 1 (II)

**Thm 1** The following are equivalent:

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Idea of proof of (i) \( \Rightarrow \) (ii)

Assume \( A \) is not minimal. Say \( A_r \) is not a cocircuit

We claim that \( M[A^{r-1}] = M[A^r] \)

...
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Idea of proof of (i) $\Rightarrow$ (ii)

Assume $A$ is not minimal. Say $A_r$ is not a cocircuit
We claim that $M[A^{r-1}] = M[A^r]$
It is essentially a consequence of the lemma of Bondy and Welsh
The weak order

Let $M_1, M_2$ be two matroids on $E$. The weak order:

$M_1 \leq_w M_2$ if every independent set in $M_1$ is independent in $M_2$. 
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Extensions and the weak order

**Fact:** the set of all extensions of a matroid $M$ is a lattice under the weak order.
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Join $M_5 \vee M_6$ and meet $M_5 \wedge M_6$: 
A question

Is the set of all transversal extensions of a transversal matroid also a lattice under the weak order?

Thus, given $N_1$ and $N_2$ two transversal extensions of $M$, is there a smallest transversal extension $N_3$ such that $N_1 \leq_w N_3$ and $N_2 \leq_w N_3$?
Example

Join $M_5 \vee M_6$ and meet $M_5 \wedge M_6$:
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Join $M_5 \vee M_6$ and meet $M_5 \wedge M_6$:

$M_5$ and $M_6$ are transversal extensions of $M$
But the ordinary join $M_5 \vee M_6$ is not transversal
Transversal join $M_5 \vee M_6$ and meet $M_5 \wedge M_6$:

$M_5$ and $M_6$ are transversal extensions of $M$.
But the ordinary join $M_5 \vee M_6$ is not transversal.
One (predictable?) result

**Thm**
For a transversal matroid $M[\mathcal{A}]$, the set of extensions obtained by adding a new element $x$ to some of the sets in $\mathcal{A}$ is a lattice under the weak order.

The transversal join of $M[\mathcal{A}^I]$ and $M[\mathcal{A}^J]$ is $M[\mathcal{A}^{I\cup J}]$.

Yet their transversal meet need not be $M[\mathcal{A}^{I\cap J}]$ if $\mathcal{A}$ is not minimal.
Wrapping up

- We know how to get all transversal extensions of a transversal matroid
- We know some ways in which repetitions can arise
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- We know some ways in which repetitions can arise.
- We would like to get each extension only once!
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- We would like to understand it better!
Wrapping up

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  We know some ways in which repetitions can arise
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- We understand a little the structure of the set of transversal extensions
  We would like to understand it better!

- We would like to use our results!